# Evolution of ultrasound beamformers

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# EVOLUTION OF ULTRASOUND BEAMFORMERS

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## ABSTRACT

Real-time ultrasonic imaging systems have been available for more than thirty years. During this time much has occurred to the basic architecture and functions of these clinical systems and their beamformers, which are, in many ways, the most important components of these systems. This talk will review some of the changes that have occurred and will discuss current trends in beamformer design.

Throughout most of the 30 years of real time imaging, analog beamformers have been the mainstay of all instruments. The common methods for the implementation of analog beamformers for annular, linear/curved arrays, and phased arrays will be reviewed and the distinguishing characteristics identified.

At the present time the industry is undergoing a major shift toward digital beamformation with the introduction of several commercial systems. Given that the earliest digital systems were available roughly 15 years ago, it is fair to say that the introduction of digital beamformers has been relatively slow. Today this shift seems to be gaining momentum. Some of the factors causing these shifts will be reviewed along the the common types of designs and implementations.

The ongoing search for improved imaging performance will continue to introduce new challenges for beamformer designers. Among already proposed imaging methods and techniques are elevation focusing (1.5D arrays), synthetic apertures, 2D and sparse arrays, phase aberration correction, and others. The most common complication introduced by these is a significant increase in channel count. Further, there will also be beamformer design issues related to signal processing. These include not only Doppler processing but also the processing of signals due to novel contrast agents. Finally, with advances in computer and microelectronics technologies, the way we view beamformation may have to undergo sizeable changes. These topics will be reviewed not only as technical challenges but also in light of the constraints introduced by today's marketplace.

## 1. INTRODUCTION

In the last 30 years, there has been considerable advancement in the functions of ultrasound systems and their beamformers. The following attempts to document the more important of these changes with a special emphasis on beamformation.

Beamformers come in a wide variety of sizes and capabilities with much of this variability coming from the different types of transducer arrays the beamformers service. The commonly used arrays are linear, curved, or linear phased arrays. The important distinctions arise from the method of beam steering used with these arrays. For linears and curvilinears, the beam steering is accomplished by selection of a group of elements whose location defines the phase center of the beam. In contrast to linear and curvilinear arrays, phased array transducers require that the beamformer steers the beam with an unswitched set of array elements. This requirement introduces significant differences in complexity over the linear and curved arrays. In spite of these differences, there is a fairly simple mathematical basis for the study of the developments that have occurred. These relations and computer simulations which illustrate the changes brought about by the beamformer evolution form the body of the paper.

A brief comment should be made about the simulations used to generate the illustrations for this paper. All the beamprofiles were generated with an angular spectrum based simulator; these are all CW models. The arrays are modeled as 2D pressure sources. The pitches, f-numbers, and focal settings are not intended to model any commercially available device and it is unlikely that the devices depicted would make strong competitive entries. The major purpose has been that of illustration of the development of beamformers over the years.

The paper is organized as follows. Section 2.0 describes the functions of a generic beamformer. The functions performed by a beamformer are reviewed and a generic block diagram is given. The basic beamformation equation will be used to identify those components which have been modified during the development process.

Section 3.0 gives a chronological account of the development of typical beamformer functions as they might be implemented with analog electronics. Heavy use is made of graphical depiction of ultrasound beamshapes to describe the influences of the various changes. Some of the different signal processing schemes are described.

Section 4.0 briefly describes the nature of digital implementation of the typical beamformer functions. The relation of these to the earlier analog implementations will be given and some of the unique challenges associated with digital beamformation will be described.

Section 5.0 gives a description of one of the newer features in many beamformers, namely that of the ability to acquire more than one acoustic line from a single transmission. This is an important step leading toward greater understanding of some of the methods synthetic aperture methods will be perfected.

Given the trends in beamformer development, Section 6 will propose a "gold standard" beamformer with respect to which the older designs can be compared to. Methods that may lead us toward the gold standard will be proposed. In view of this goal, Section 7
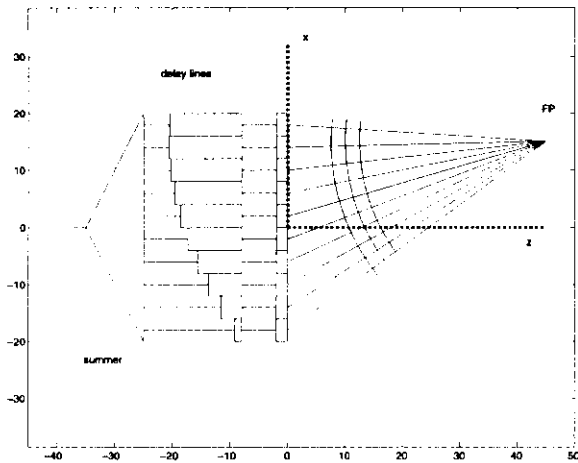
Figure 1: Basic geometry for beamformation calculations

describes approaches using synthetic aperture methods.

Finally, Section 8 describes some of the work in the area of transducer arrays leading toward 2D arrays. Some of the implications to beamformer design will be given.

## 2. BEAMFORMER BASICS

The functions of a beamformer include the following:

- generate transmit timing and possible apodization (the term apodization will be used as a synonym for weighting, tapering, and shading) during transmit

- supply the time delays and signal processing during receive

- supply apodization and summing of delayed echoes

- possible additional signal processing related activities

The goal of all of these functions is to create as narrow and uniform a beam with as low sidelobes over as long a depth as possible. During both transmit and receive operations, appropriate delays will be supplied to accomplish the focusing and steering needed. Figure 1 demonstrates the geometry that is usually used with most discussions on this topic. Figure 1 illustrates the reception process; wavefronts are shown emanating from a point source labeled as FP. These signals are received by the array elements, amplified, and passed on to the delay lines. The delay lines are shown as rectangular boxes whose length corresponds to the desired delay. Finally, the echoes are passed on to the apodization/summer stage, which takes the contributions from each element, multiplies them with a weighting function, and adds up the results. The transmit operation is essentially the inverse of receive focusing; time delays from a common synchronization signal are generated by some means, often down counters, and the array elements are fired accordingly. It is assumed the array elements act as point sources and generate the required wavefronts much as implied by Figure 1. A good number of the references listed below discuss the transmit and receive operations in greater detail than is possible here [2, 18, 27, 46, 48, 49, 51, 42, 55].

In this paper, we will be studying beamformer functions with the help of the following expression for the received echo $r(t)$:

$$r(t) = \sum_{i=1}^{N} A_{ri} \sum_{j=1}^{N} A_{xj} s(t - \tau_{ri} - \tau_{xj} + \frac{2R_{fp}(t)}{c}). \quad (1)$$

In this expression, the transmitted waveshape is s(t). The A's refer to whatever weighting functions that might be applied to each of the channels during the transmit and receive operations. In the simplest case these would be equal to one for uniform aperture weighting. Similarly, the $\tau$'s refer to the transmit and receive delays applied during transmit and receive beamformation operations. $i$ and $j$ are indices of the receive and transmit elements, respectively, and subscripts $r$ and $x$ refer to receive and transmit operations. These four parameters, $A_{ri}$, $A_{xj}$, $\tau_{ri}$, and $\tau_{xj}$ will form the basis of the discussion on beamformer evolution; changes in their values and in the methods by which their role has been implemented has defined the different generations of instruments. Quality of beamformation is strongly influenced by them.

Finally, in Equation 1, $N$ is the number of transmit and receive elements and will be assumed to be constant in the following. $N$ is also an important factor in establishing the performance level and the cost of an ultrasound system. However, its role in the beamformation process will be only considered indirectly in the following.

Referring to Figure 1, the expression for determining the values for the transmit and receive delays given a desired focal point is:

$$\tau_i = \frac{1}{c}[\sqrt{(x_i - x_{fp})^2 + z_{fp}^2} - R_{fp}]. \quad (2)$$

In this expression, c is the speed of sound, $\tau_i$ is the transmit or receive delay, $x_{fp}$ and $z_{fp}$ are the coordinates of the point at which we wish to focus, and $R_{fp}$ is the distance from the origin or the phase center to that focal point. The focal point may be one of the following:

- a fixed point in space such as a transmit focal point

- a point which moves at the speed of sound with the wavefront of the transmitted pulse as in dynamic focusing

- a point corresponding to a pixel in an image to be formed as with a synthetic aperture approach.

These cases will be discussed as they arise in the following sections.

## 3. BEAMFORMER DEVELOPMENT

It is instructive to review the functions of a beamformer from the impact these functions have on the beamshape. In this section we will review the major evolutionary steps in beamformer development from this vantage point. We will start from the very early designs and work our way toward the present.

The earliest phased array beamformers were developed in the late 1960's for imaging of the brain [44] and in the early 1970's for echocardiography [2, 49, 51]. Linear array systems were initially developed for echocardiography [5, 20] and for obstetrics and gynecology. These early systems involved relatively simple implementations of the beamformer functions [2, 44]. With some of the linear array designs, no focusing was included; they relied completely on using a collimated beam and the narrowing of the beamshape at the near-to-far field transition. Figure 2 shows a transmit beamshape that might be generated from a fixed focus
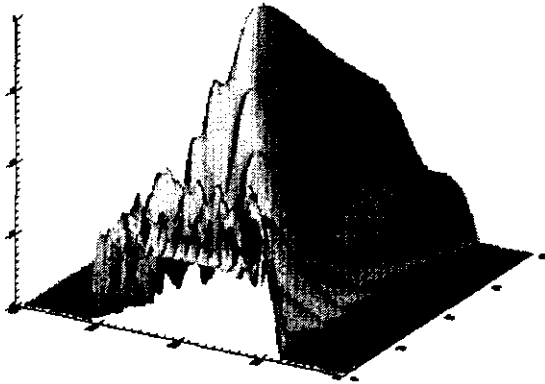
Figure 2: Transmit beamshape, fixed focus



Figure 3: Transmit beamshape, fixed focus, apodization

system. Several significant limitations are immediately apparent. For example, the focal region is quite limited, the side lobe levels are quite high, and the near field extensive. The transmit only beamformer equation is given below:

$$r(t) = \sum_{j=1}^{N} A_{xj} s(t - \tau_{xj} + \frac{2R_{fp}(t)}{c}). \qquad (3)$$

The limited focal region is directly associated with the use of constant values for the $\tau_{xj}$ in Equation 1. As a consequence the $\tau_{xj}$ will cancel the $\frac{2R_{fp}(t)}{c}$ term only in one location. Further, what may not be apparent in Figure 2 is the need to use a fairly high f-number so that the depth of field is acceptable for routine imaging. It is likely that this and the points made earlier about the beamshape itself the major reasons why the market acceptance of real-time systems was fairly slow; the articulated-arm contact B-scanners remained dominant well into the late 1970's.

The need to suppress sidelobes in clinical imaging was recognized fairly early on [11]. Figure 3 shows the impact of apodization on the beamshape generated by the aperture in Figure 2. The combination of transmit and receive beamformation does lower sidelobe levels considerably, this performance can be dramatically improved by the introduction of apodization or weighting of the transmit pulses and/or received echoes by an appropriate weighting functions (see Figure 3). Reports on this came out in the literature in the late 70's [11, 18, 48, 55]. These processing steps became available in commercial instruments starting from about 1980 and onwards. Along with dynamic apodization came the capability to increase the aperture size dynamically during receive, i.e. dynamic aperture.

The concept of dynamic focusing with medical ultrasound transducers goes back to the 1950's [41], however, its reduction to practice required about 20 years. The first Duke University phased array system [49, 51] was capable, at least in principle, of it and reports of designs were reported in the literature in the late 70's and early 80's [7, 18, 29, 52]. Figure 4 shows a beamshape with the introduction of dynamic receive focusing. A major benefit from dynamic focusing is the ability to lower the f-number during receive beamformation and to keep it relatively constant until one runs out of aperture. This was of obvious value with high
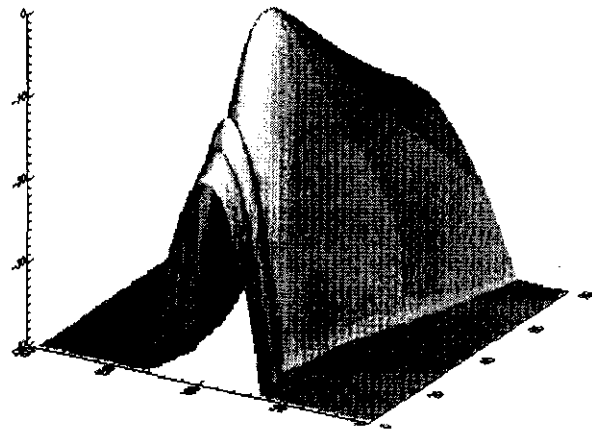
channel count systems. Implementation of dynamic focusing in analog beamformers is usually accomplished by the use of coarse and fine delay circuit blocks. In a typical design the coarse delay was realized by the use of a summing delay line which was preceeded by a cross-point switch to select the appropriate tap for a particular channel. Two types of fine delay implementation have been reported:

1. by heterodyning the received signal to baseband or an intermediate frequency with a mixing signal whose phase was changed as needed for dynamic focusing [7, 18, 29]

2. by choosing a tap of a shorter delay line by a very carefully designed switch or switches [2, 49].

The heterodyning approach is more of a narrowband approach in that the fine delay correction is valid over a limited frequency range [47]. However the phase and hence time delay control that is achievable with that method is very good. The debate on this issue, which has carried on to digital beamformation, is likely to last a long time.

There is considerable subtlety to the implementation of the required delays and the selection of delay quanta and other parameters. A sizeable body of literature exists on the impact of amplitude and phase quantization on sidelobes as well as grating lobes [3, 4, 28, 32, 34, 40, 46, 47]. Much of the cost variation associated with different types of systems is related to the extent to which the system designers have attempted to minimize such errors. As will be discussed later, the entry into digital beamformation has required a re-evaluation of many of these issues.

Interestingly, while the implementation of beam steering with linear and curvilinear arrays is far easier than with phased arrays, it turns out the application of dynamic apodization is considerably more complex. The reason for this has to do with the fact that with an expanding aperture the active aperture is very likely to reach the edges of the physical aperture. The implementation of this turns out to be highly complex due to the large number of possible operating conditions and transducers. The optimum design of an apodization curve for a truncated aperture is also not immediately obvious.

The initial clinical application areas did not always turn out to be the final choices for the physicians. The linear and later
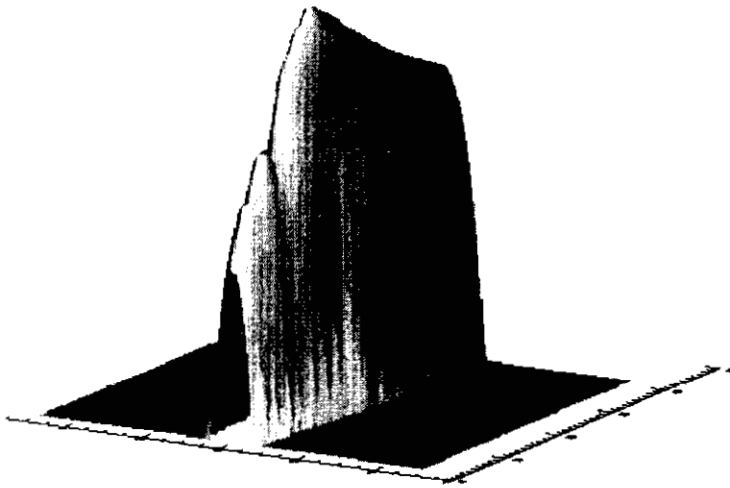
Figure 4: Transmit/receive beam with dynamic focusing, apodization

curvilinear arrays became the choices for the radiologist for general imaging applications and for obstetrics and gynecology. The linear array based systems could be built for low cost relative to phased arrays since there was no need for long delay lines necessary beam steering and the apertures could be realized with fewer elements (there was no need for $\lambda/2$ pitch in the arrays). This low cost made linear array systems a very popular choice for the private office clinicians such as ob/gyn specialists. Perhaps due to their small contact area with the patients and the requirement for a very narrow imaging window, phased arrays became the primary tool for the echocardiographer. This specialization has introduced several variations to the beamformer designs.

## 4. DIGITAL BEAMFORMATION

While the earliest commercially available digital beamformers were available in the early 80's, they did not begin to have a significant impact until the early 90's. Much of this delay was due to the need for A/D converters with sufficiently large number of bits and a high enough sampling rate. Another factor which has facilitated this change is the dramatic increases in gate counts of ASIC's and the improvements in their design tools. This is reflected by some of the designs or discussions of various designs published in the literature [13, 15, 16, 19, 21, 27, 30, 36, 37, 40, 45, 47]. An obviously important topic is the number of bits required for A/D conversion. The literature does not supply a crisp answer; most expressions given are contrained by the assumptions made. Peterson et al. [40] give the quantization induced sidelobe level as $1/2^B\sqrt{3N}$, while Steinberg [47], who considers the possibility of partial coherence in the quantization noise, describes the same quantity as $-10logN_{eff} - 6B$. In both expressions $B$ is the number of bits, $N$ is the number of processing channels, and $N_{eff}$ is the number of statistically independent channels. With respect to sampling rate, it is, of course, important to sample at a rate sufficient to capture all of the information in the array bandwidth. There are the usual issues with selection of appropriate cut-off frequencies for the anti-aliasing filters. The large channel counts of today's systems tend to force a relatively simple filter design; higher sampling rates might be helpful to enable one to perform additional bandpass filtering in
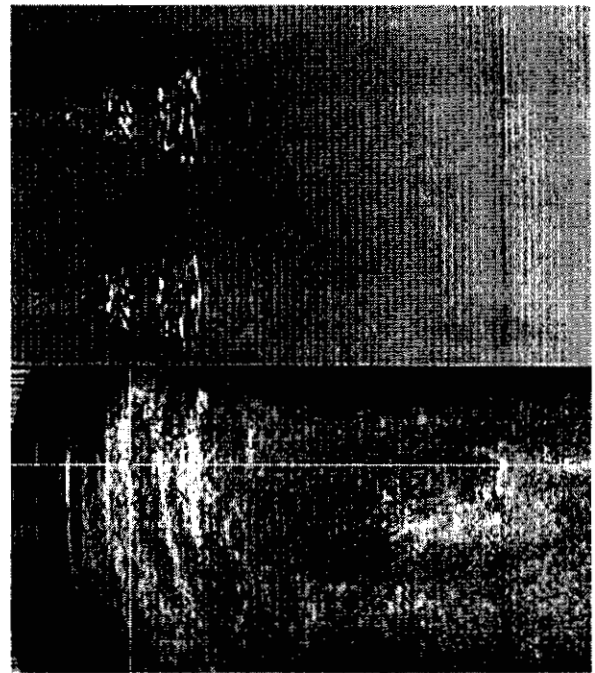


Figure 5: In-phase and quadrature signals

the digital domain.

Today the rate of transfer to digital processing is gaining momentum, although due to some of the costs associated with the required components, most of the digital systems tend to be high end machines. However, given the potential cost reductions of the key components and some clever algorithmic designs, some of this may change [15, 17, 19, 45].

Implementation of apodization and delay functions in digital beamformers have followed the approaches used with their analog counterparts, i.e. the architectures have been of the delay-and-sum type and by the use of combinations of coarse and fine delays for the delay structures. Fine delays have been implemented by heterodyning and by interpolation. However, given the flexibility that digital beamformation offers and and the ability to store a sufficiently high dynamic range of the echo information, the potential to experiment with novel beamformation approaches certainly exists.

## 5. MULTI-LINE ACQUISITION

Figure 5 illustrates several interesting points in the area of beamformation. The top section of this figure shows an image of the in-phase and quadrature (I and Q) components of an echo signal for each of the elements of an array acquired in an *in vivo* situation. The dimensions of the figure are depth into the body horizontally and channel number vertically (repeated for both components). The bottom part of the figure shows the corresponding B-mode image from which the I and Q plots were derived along with the cursors to show the location from which the I and Q data originate. These data give the I and Q components before apodization (and, of course, summing) but after the steering and focusing delays have been applied. The two images are sector shaped because of an

expanding aperture capability which has been applied.

In an I and Q data set such as this, echoes which are in phase will form a vertical line in both images. Thus as the summer stage adds up the contributions from the elements, they will add up constructively. This can be seen in these images in the far field where echoes from a strong scatterer (visible in the B-mode under the cursor line) show up as nearly vertical lines in the image. It is immediately apparent that at this depth, the steering and focusing operations are likely to yield good performance. There is a direct contrast to this in the near field of the I and Q images. The I and Q pattern here is far more complex implying the existence of echo sources from multiple different directions. In the near field, the transmit pulse has not yet formed a tight beam, hence there are numerous possible sources for these echoes. This is also true with respect to the elevation plane; arrays such as this one usually have a fixed focus such as a lens to concentrate the beam. It might be mentioned that from what is known at this point, it is not clear whether the echo-rich pattern shown is due to sources in the image plane or from the elevation.

Given the availability of additional information which would be otherwise lost, it was recognized that one could steer additional receive beams in those directions which were sonified by the transmit beam and to acquire multiple beams from a single transmission. This will result in some loss in beam performance since the full impact of the transmit and receive beamformation operations will not be available. Further there will likely be some "beam wander" in the direction of the resulting beam due to the effects of varying transmit beamwidth, but this usually can be minimized. Finally, there may be some loss in penetration due to the need to transmit a slightly defocused beam. Nonetheless, this mode of operation has found an important niche in an application such as color Doppler imaging, where beam shape is not critical and where frame rates are of great importance.

## 6. "GOLD STANDARD" BEAMFORMER

The beamformation process often considered the best possible is the one associated with image formation from a complete data set (or the $N^2$ data set) [12, 17]. Since acquisition of such a data set will require roughly $(N^2)/2$ transmits and receives (on the assumption that reciprocity holds), this approach is clearly not feasible in an in vivo setting. Equation 1 can be easily modified to depict this type of data acquisition:

$$r(t) = \sum_{i=1}^{N} A_{ri}(t) \sum_{j=1}^{N} A_{xj}(t) s(t - \tau_{ri}(t) - \tau_{xj}(t) + \frac{2R_{fp}(t)}{c}).$$

As can be seen, all beamforming parameters are functions of time (or, equivalently, depth) including the transmit focusing and apodization terms. The result is that one can realize both apodization and dynamic focusing both on transmit and receive and for each pixel of the image. Not surprisingly, excellent beamwidth and other characteristics can be achieved. Figure 6 is a beamshape using the parameters from the prior simulations but with gold standard processing. The goal of much of the beamformer development over the last 20 years has been to begin to approach this level of performance.

There are several methods that one can use to approximate the "gold standard" performance with differing amounts of potential for reaching it. Some of these are listed here:
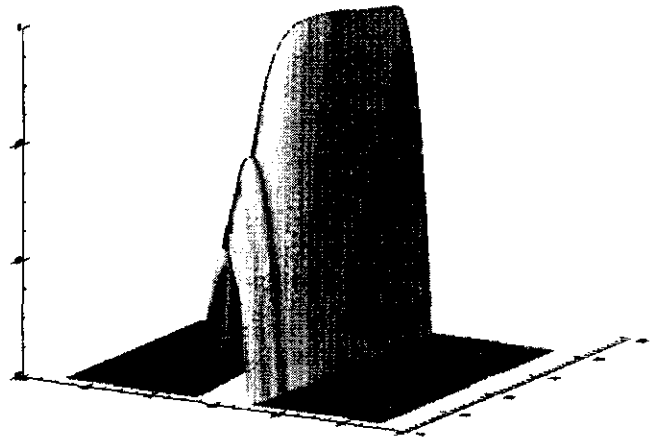


Figure 6: Gold standard beamformer

1. multiple transmits at sequential focal locations
2. synthetic aperture approaches
3. non-diffracting transmit beam [26]
4. deconvolution of transmit beam pattern [12]

The first of these options clearly does allow considerable flexibility in the assignment of the $\tau$ and $A$ values of Equation 1 although this will occur at the expense of frame rate. In this manner the image data acquisition can be optimized for most depths. Strong application areas are likely to be those in which the pulse repetition frequency can be very high. The second option will be discussed in greater detail in the next section. The third choice involves an elegant solution to the wave equation in which one can generate a pencil like beam over most of the range of interest. Unfortunately, this beam uniformity is achieved at the expense of very high sidelobes. Thus, some resolution will have to be sacrificed in an implementation of such systems since the receive apodization will have to be relied upon to suppress them. The last choice may have a good amount of potential especially if a sufficiently simple real-time implementation can be developed.

## 7. SYNTHETIC APERTURE DESIGNS

In discussing beamformation with synthetic apertures, it will be useful to separate the synthetic approaches into two classes:

- synthetic receive apertures
- synthetic transmit or transmit/receive aperture

With synthetic receive processing one goal might be to achieve a level of performance without actually having the full number of real receive channels. The approach used involves transmitting with a full aperture and receiving with two or more subarrays. With respect to issues such as penetration and the quality of beamformation, this is a very robust technique. A weakness with this (as well as other synthetic aperture methods) approach arises from tissue motion; for example, synthetic aperture processing is unlikely to work well with cardiac imaging. More specifically with respect to synthetic receive processing, the additional transmits that are required with this approach will not bring about any additional

processing gain other than the ones achieved by the extension of the receive aperture.

In contrast with synthetic receive aperture processing, synthetic transmit aperture involves transmitting from *two or more* transmit subapertures (with synthetic receive, the multiple transmissions were always done with a full aperture). This approach has received much attention in the literature although few if any *in vivo* results have been reported [1, 6, 13, 17, 22, 33, 34, 35, 38, 39, 56]. An exception to this is the area of intra-arterial imaging where some significant success has been realized [22, 38, 39]. In further contrast with synthetic receive processing, it is now possible that both larger transmit <u>and</u> receive apertures can be realized and, further, that steps can be taken to include the ability to vary the transmit focus dynamically thereby extending the focal area.

Equation 1 has to be modified for this type of processing to include the multiple transmissions:

$$r(t) = \sum_{k=1}^{L} A_k \sum_{i=1}^{N} A_{ri}(t) \sum_{j=1}^{N} A_{xkj}(t) \times$$
$$s(t - \tau_{rik}(t) - \tau_{xkj}(t) + \frac{2R_{fp}(t)}{c}). \qquad (4)$$

The additional transmits permit additional degrees of freedom in setting the weighting factors and time delays for the now increasingly independent transmit and receive operations. With a digitally stored data set it is also possible to introduce pixel-specific focusing with these additional degrees of freedom.

There are limitations to the synthetic transmit aperture beamformation method related to penetration of ultrasound in tissue. There will be some loss of echo strength associated with beam spreading since the transmit beam will not be as well focused as with methods which use the full transmit aperture. Indeed, the parameter $L$ in Equation 4, which describes the number of independent transmits contributing to the beamformation and which one would like to maximize, is inversely related to the quality (i.e. beamwidth) of the transmit beam. In other words, the wider the transmit beam, the more independent contributions are made to the beam sum in Equation 4. The synthetic transmit aperture method is likely to work well in those regions where there are no dynamic range limitations. Another possible limitation depends on the implementation methods selected by the designers. If the in-phase and quadrature data are used, the performance will be that of a more narrowband system. Use of wider bandwidth signals in this context results in a considerable increase in complexity. These tradeoffs will have to be carefully considered. Nonetheless, there are attractive operating regions for this approach.

There are a number of implications to beamformer implementation from this type of processing. For example, the beamformer will have to be fast enough to be able to process the multi-beam data and to pass it on to the scan converter. This does imply a considerable shift in the beamformer function from a basic delay-and-sum role to a more complex beamforming computer.

## 8. IMPACT OF DIFFERENT TYPES OF 2D ARRAYS

As noted earlier, the present generation of ultrasound arrays has a fixed mechanical lens to concentrate the beam in the elevation direction. As a consequence, the beam profile in the elevation direction has considerable similarity to the one shown in Figure 2 with all its shortcomings such as limited focal depth, poor resolution, etc. This problem is receiving increasing amount of attention at the present time [9, 43, 50, 54]. It may be useful to discuss some of the terminology that has been used around these developments. The following nomenclature has been described in the literature for arrays between simple linears and full 2D arrays:

- 1.25D arrays: multi-row arrays with expanding aperture capability,

- 1.5D arrays: multi-row arrays with electronically controlled focusing in the elevation but with delay symmetry along the center line,

- 1.75D arrays: multi-row arrays with independently controlled delays for all rows in the array,

- 2D arrays: fully (or nearly so) sampled array with full connections.

In the near future, configurations in the first two categories are likely to see implementation; the last two are longer term developments. The 1.25D arrays are unlikely to require any significant change in beamformer implementation. The 1.5D arrays will require sufficient channels to process the additional signals but little new as far as echo processing goes. The 1.75D arrays have been considered to be useful for phase aberration corrections [54] and hence will require whatever capability is required to implement that capability. The first 2D implementation will take advantage of sparse array technology discussed in the next paragraph.

Due to limits in transducer array and interconnect technology as well as limitations in economically feasible channel counts, investigators have begun looking into beamformation with sparse arrays [8, 10, 14, 23, 24, 43, 53]. One likely impact of these on beamformer design will be that of channel count. It is probable that the performance of sparse arrays will be somewhat worse than that of traditional filled arrays. The important issue will be the additional benefit one can gain from the 2D nature of data acquisition which will be possible with such devices.

An important area that has not received much attention in the discussion of sparse array beamformer design is that of processing channel uniformity. While with either the co-array or effective aperture syntheses [8, 14, 23, 24, 25] one can design arrays which take advantage of the inherent redundancy of traditional image data acquisition. It is not clear to what degree the current systems rely on this redundancy to average out variations which arise from the processing channels, element performance, and the influences due to signal path nonuniformity.

## 9. CONCLUSIONS

There is little doubt that the activity level in beamformer development will remain at a very high level. Not only will the number of systems with digital beamformers increase but the application of that technology to lower cost systems will, most likely, come about. The flexibility of digital beamformers will permit novel beamforming methods to be exploited. Areas such as elevation focusing, synthetic apertures, and others are currently under development and should enter into commercial instruments in the near future (if they haven't done so already).

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